Hoofdstuk 1: Vectorrekening

① $(\vec{A} \times \vec{B})^2 = \vec{A}^2 . \vec{B}^2 - (\vec{A}.\vec{B})^2$

uitkomst = scalair

$\vec{A}^2 = |\vec{A}|^2$ = getal

identiteit van Lagrange

$$LL = \left| |\vec{A}| . |\vec{B}| \sin\theta \right|^2$$
$$= |\vec{A}|^2 . |\vec{B}|^2 . (1 - \cos^2\theta)$$
$$= |\vec{A}|^2 . |\vec{B}|^2 - (\vec{A}.\vec{B})^2$$
$$= \vec{A}^2 . \vec{B}^2 - (\vec{A}.\vec{B})^2$$

② $\vec{V}$ = ?

$\vec{A}.\vec{V} = p$ $\qquad |\vec{A}|.|\vec{V}|.\cos\theta = p$

$\vec{A} \times \vec{V} = \vec{B}$ $\qquad |\vec{A}|.|\vec{V}|.\sin\theta = |\vec{B}|$

$$p^2 + |\vec{B}|^2 = |\vec{A}|^2 . |\vec{V}|^2 \, \cancel{(\sin^2\theta + \cos^2\theta)}$$
$$|\vec{V}| = \sqrt{\frac{p^2 + |\vec{B}|^2}{|\vec{A}|^2}} = \frac{\sqrt{p^2 + |\vec{B}|^2}}{|\vec{A}|}$$

want $\vec{A} \neq \vec{0}$: OK

③ $\vec{e}_x \times (\vec{V} \times \vec{e}_x) + \vec{e}_y \times (\vec{V} \times \vec{e}_y) + \vec{e}_z \times (\vec{V} \times \vec{e}_z) = 2\vec{V}$

$\cancel{LL = \vec{e}_x \times (|\vec{V}| . \sin\theta \, \vec{e}_x) + \vec{e}_y \times (|\vec{V}| . \sin\theta \, \vec{e}_y) + \vec{e}_z \times (|\vec{V}| . \sin\theta \, \vec{e}_z)}$

$\boxed{\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\,\vec{B} - (\vec{A}.\vec{B})\,\vec{C}}$ (theorie)

oplossing = vector

$(\vec{e}_x . \vec{e}_x)\,\vec{V} - (\vec{e}_x . \vec{V})\,\vec{e}_x$

$= \vec{V} - V_x . \vec{e}_x$

$\dots \vec{V} - V_y \vec{e}_y$

$\dots \vec{V} - V_z \vec{e}_y$

$$3\vec{V} - \underbrace{(V_x \vec{e}_x + V_y \vec{e}_y + V_z \vec{e}_z)}_{\vec{V}} = 2\vec{V}$$

2e manier:

$\vec{e}_x \times (\vec{V} \times \vec{e}_x)$

$$\vec{V} \times \vec{e}_x = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ V_x & V_y & V_z \\ 1 & 0 & 0 \end{vmatrix} = V_z \vec{e}_y - V_y \vec{e}_z$$

$$\vec{e}_x \times (\vec{V} \times \vec{e}_x) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ 0 & V_z & -V_y \end{vmatrix} = -\vec{e}_y(-V_y) + \vec{e}_z(V_z)$$

$= V_y \vec{e}_y + V_z \vec{e}_z$

$= V_x \vec{e}_x + V_z \vec{e}_z$

$= V_x \vec{e}_x + V_y \vec{e}_y$

$+$

$$2V_x \vec{e}_x + 2V_y \vec{e}_y + 2V_z \vec{e}_z = 2\vec{V} \quad \square$$

⑤ $\vec{V_1} = (1,0,1)$ i) gericht vectorproduct:

$\vec{V_2} = (0,1,1)$ scalair product v/e vectorproduct

$\vec{V_3} = (-1,-1,1)$ $\vec{A}\,.\,(\vec{B}\times\vec{C}) = 0$ = lineair onafhankelijk

$$V_1\,.\,(V_2 \times V_3) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \dots = 3 \neq 0$$

ii) onderling loodrecht : $\vec{A}\,.\,\vec{B} = 0$

scalair product

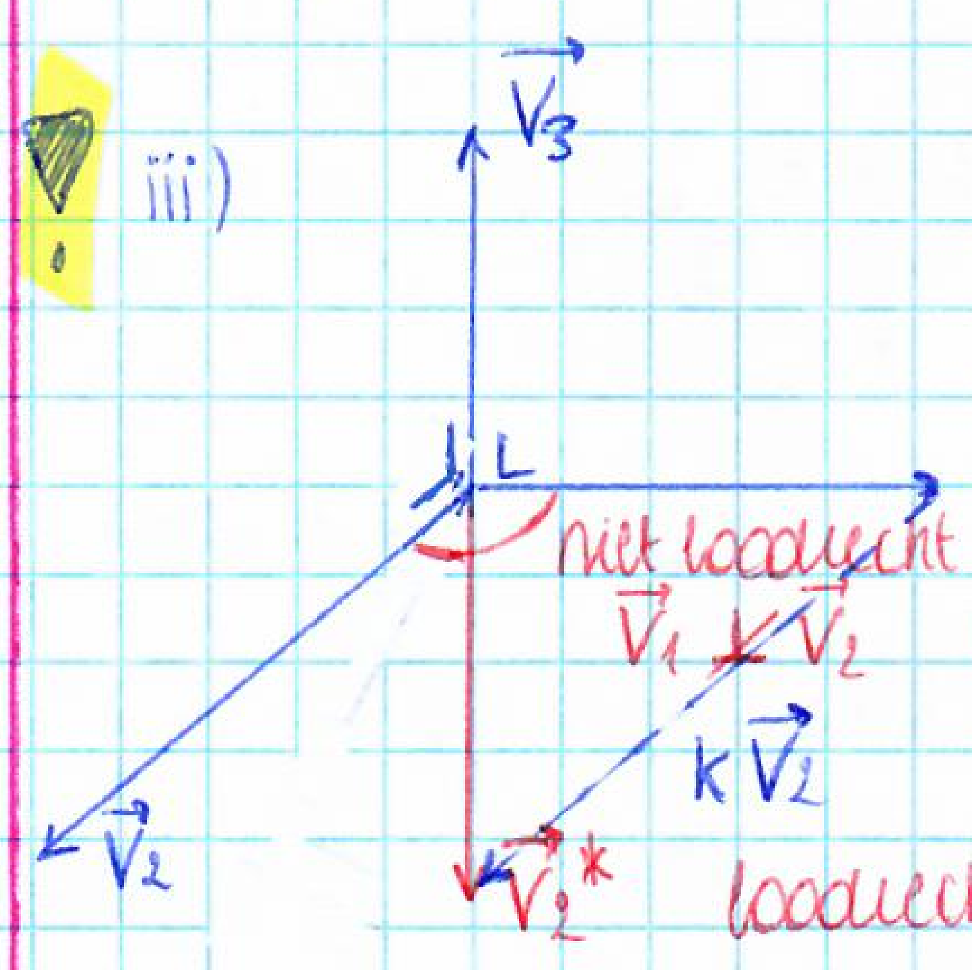
$\vec{V_1}\cdot\vec{V_2} = 1.0 + 0.1 + 1.1 = 1$ $\quad \vec{V_1} \not\perp \vec{V_2}$

$\vec{V_1}\,.\,\vec{V_3} = 1.-1 + 0.-1 + 1.1 = 0$ $\quad \vec{V_1} \perp \vec{V_3}$

$\vec{V_2}\,.\,\vec{V_3} = 0.-1 + 1.-1 + 1.1 = 0$ $\quad \vec{V_2} \perp \vec{V_3}$

uitleg einde vd cursus lezen!

! iii)



we zoeken een nieuwe $\vec{V_2}^*$ die samen met $\vec{V_1}$ & $\vec{V_3}$ het orthogonaal assenstelsel maken

loodrecht op $\vec{V_3}$ & $\vec{V_1}$ $\qquad k$

$$\vec{V_2}^* = \vec{V_1} + k\vec{V_2}$$

$$\vec{V_1}\,.\,\vec{V_2}^* = 0 = \vec{V_1}\cdot(\vec{V_1} + k\vec{V_2})$$

k?

$$k\,V_2 V_1 + + \vec{V_1}\vec{V_1} = 0$$

$$k + 2 = 0$$

$$\boxed{k = -2} \qquad \vec{V_2}^* = \vec{V_1} - 2\vec{V_2} = (1,-2,-1)$$

vervolg : normaliseren van de vector

$$\begin{cases} \vec{u_1} = \dfrac{\vec{V_1}}{\|\vec{V_1}\|} = \dfrac{\vec{V_1}}{\sqrt{1^2+0+1^2}} = \dfrac{\sqrt{2}}{2}(1,0,1) \\[2ex] \vec{u_2} = \dfrac{\vec{V_2}^*}{\|\vec{V_2}^*\|} = \dfrac{\sqrt{6}}{6}(1,-2,-2) \\[2ex] \vec{u_3} = \dfrac{\sqrt{3}}{3}(-1,-1,1) \end{cases}$$

of via

$(\vec{V_2}, \vec{V_3}, \vec{V_2}\times\vec{V_3})$

$(\vec{V_1}, \vec{V_3}, \vec{V_1}\times\vec{V_3})$

$\boxed{2^e \text{ manier}}$ orthogonaal stel $(\vec{V}_1, \vec{V}_3, \vec{V}_1 \times \vec{V}_3)$

$$\vec{u}_3 = \frac{\vec{V}_1 \times \vec{V}_3}{\|\vec{V}_1 \times \vec{V}_3\|} = \frac{\vec{V}_1 \times \vec{V}_3}{|\vec{V}_1||\vec{V}_3| \underbrace{\sin\theta}_{1}\ \frac{\pi}{2}} = \frac{\vec{V}_1}{\|\vec{V}_1\|} \times \frac{\vec{V}_3}{\|\vec{V}_3\|} = \vec{V}_1 \times \vec{V}_3$$

want $\vec{V}_1 \perp \vec{V}_3$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \begin{vmatrix} e_x & e_y & e_z \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \frac{\sqrt{6}}{6} (2, -1, -1)$$

$$U_1 = \frac{\vec{V}_1}{\|\vec{V}_1\|} = \frac{\sqrt{2}}{2} (1, 0, 1)$$

$$U_2 = \frac{\vec{V}_3}{\|\vec{V}_3\|} = \frac{\sqrt{3}}{3} (-1, -1, 1)$$